Let set P = { 000, 001, 010, 011, 100, 101, 110, 111} and let the partial order x ≤ y be defined as:
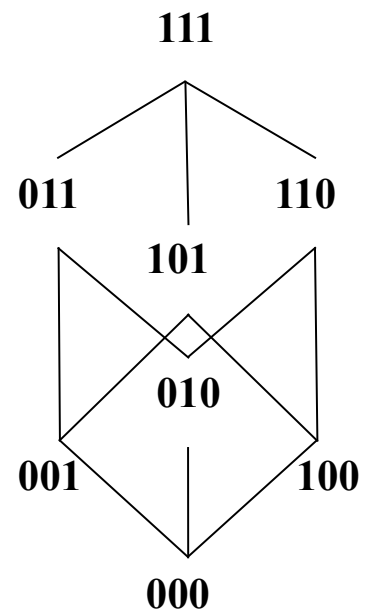
   x ≤ y if (x & y) = x   where & denotes bitwise AND

The figure on the right shows the Hasse diagram of the lattice (P, ≤).

1. What is the least upper bound of the set {101, 100, 001}?

   101

2. What is the greatest lower bound of the set {101, 100, 001}?

   000

3. Is this a complete lattice? If it is a complete lattice, describe how to compute the least upper bound and greatest lower bound of any subset of P. If it is not a complete lattice, specify a subset of P for which either a least upper bound or a greatest lower bound does not exist.

   The lattice is complete. For any subset of P, its least upper bound can be computed as the bitwise-OR of all elements in the subset, while its greatest lower bound can be computed as the bitwise-AND of all elements in the subset.